ON MASS TRANSFER IN AN ACOUSTIC FIELD

A. P. Burdukov and V. E. Nakoryakov

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Chemical processes governed by the laws of diffusion kinetics can be intensified by elastic oscillations. It is also known that the rate of combustion of liquid and solid fuels changes markedly with the onset of acoustic vibrations in the combustion chamber. Despite the extensive application of vibrational processes in technology, the mechanisms of heat and mass transfer in the presence of vibrations are not well known. The aim of this research was to analyze the mass transfer from a sphere in an acoustic field.

Notation

ω – angular frequency of oscillations,
λ – wavelength,
R – characteristic dimension of axisymmetric body,
s – amplitude of displacement of fluid particles in a plane acoustic wave,
B – amplitude of oscillation velocity,
x, y – longitudinal and transverse coordinates,
u, v – longitudinal and transverse velocity components,
ν – kinematic viscosity,
U – $A(x)\cos\omega t$ – velocity of potential flow,
δ^+ – thickness of momentum boundary layer,
δ^- – thickness of diffusion boundary layer,
m – dimensionless concentration,
m_* – concentration of diffusing component at surface of vaporization;
t – time;
D – diffusion coefficient
ρ – average density of mixture;
erf – error function;
r – radius of axisymmetric body;
R – Reynolds number;
P – diffusion Prandtl number;
<> – time average;
N, N_d – Nusselt numbers based on radius and diameter, respectively;
' – pulsating component of velocity or concentration;
$_0$ – stationary component of velocity or concentration.

1. Consider an axisymmetric body immersed in a fluid perturbed by a plane acoustic wave. The main assumptions are as follows:

a. The density of the mixture is constant. The velocity field is independent of the concentration field.

b. The viscosity of the gas and the diffusion coefficient are independent of the concentration field.

c. The body is perfectly rigid.

d. The wavelength of the acoustic oscillations is much longer than the characteristic dimension of the body ($\lambda/R \gg 1$), which makes it possible to regard the fluid near the wall of the body as incompressible.

e. The oscillatory Reynolds number $\omega R^2/\nu$ is sufficiently large, i.e., the boundary-layer equations may be used.

With these assumptions one may distinguish two limiting cases – when the amplitude of particle displacement is much larged ($s/R \gg 1$) or much smaller ($s/R \ll 1$) than the dimension of the body.

2. Let us analyze the hydrodynamics of the process for the case when $s/R \ll 1$. Experimental investigations [2, 3] show that in this case there is a steady secondary flow near the body. With the above assumptions, the equations of continuity and momentum and the boundary conditions in the oxy coordinate system (Fig. 1) are

$$\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = \nu\frac{\partial^2 u}{\partial y^2} + \frac{\partial U}{\partial t} + U\frac{\partial U}{\partial x}, \qquad \frac{\partial (ur)}{\partial x} + \frac{\partial (vr)}{\partial y} = 0, \tag{2.1}$$

$$u = U = A(x)\cos\omega t \quad \text{at} \quad y = \infty, \qquad u = v = 0 \quad \text{at} \quad y = 0, \tag{2.2}$$

This problem can be solved by the method used by Schlichting [4] in the analysis of the boundary layer on a cylindrical body. Let us represent the velocity field as a sum $u = u' + u_0$.

With the assumption $s/R \ll 1$, the equations governing the pulsating component of velocity are

$$\frac{\partial u'}{\partial t} - \nu \frac{\partial^2 u'}{\partial y^2} = \frac{\partial U}{\partial t}, \qquad \begin{array}{l} u' = 5 \quad \text{at } y = 0, \\ u' = U = A(x) \cos \omega t \quad \text{at } y = \infty. \end{array} \tag{2.3}$$

The solution of (2.3) is

$$u' = A(x)\left[\cos \omega t - e^{-\eta} \cos(\omega t - \eta)\right] \qquad (\eta = y\sqrt{\omega/2\nu}). \tag{2.4}$$

The normal velocity component v' is determined from the equation of continuity (2.2).

The equation for the component of velocity is

$$\nu \frac{\partial^2 u_0}{\partial y^2} = \left\langle u' \frac{\partial u'}{\partial x} \right\rangle + \left\langle v' \frac{\partial u'}{\partial y} \right\rangle - \left\langle U \frac{\partial U}{\partial x} \right\rangle.$$

Evaluating the averaged terms, we obtain

$$\begin{aligned} \frac{\partial^2 u_0}{\partial \eta^2} = & -\frac{A(x)}{\omega} \frac{\partial A(x)}{\partial x} \left[e^{-\eta}(2+\eta)\cos\eta - e^{-\eta}(1-\eta)\sin\eta - e^{-2\eta}\right] - \\ & - \frac{1}{r(x)} \frac{\partial r(x)}{\partial x} \frac{A^2(x)}{\omega} \left[\eta e^{-\eta}(\cos\eta + \sin\eta) - e^{-\eta}\sin\eta\right], \end{aligned} \tag{2.5}$$

which, taking into account the boundary conditions

$$u_0 = 0 \quad \text{at } \eta = 0, \qquad \partial u_0 / \partial \eta = 0 \quad \text{at } \eta = \infty,$$

yields the result

$$\begin{aligned} u_0 = & \frac{A(x)}{\omega} \frac{\partial A(x)}{\partial x} \left[\frac{1}{2} \eta e^{-\eta}(\sin\eta - \cos\eta) + e^{-\eta}\left(2\sin\eta + \frac{1}{2}\cos\eta\right) + \right. \\ & \left. + \frac{1}{4} e^{-2\eta}\right] - \frac{3}{4} \frac{A(x)}{\omega} \frac{\partial A(x)}{\partial x} - \frac{A^2(x)}{\omega r(x)} \frac{\partial r(x)}{\partial x} \left\{\frac{1}{2} - \right. \\ & \left. - \left[e^{-\eta}\left(\frac{\eta}{2} + 1\right)\sin\eta + \frac{e^{-\eta}}{2}(1-\eta)\cos\eta\right]\right\}. \end{aligned} \tag{2.6}$$

Equation (2.6) for the stationary component of the velocity differs from Schlichting's solution in virtue of the presence of the terms in braces, which are due to axisymmetry. Analysis of equations (2.5) and (2.6) shows that the momentum boundary-layer thickness is the same for the stationary and pulsating components and is equal to

$$\delta^+ = \sqrt{2\nu/\omega}.$$

Outside the mementum boundary layer, the longitudinal velocity component is given by

$$u = -\frac{3}{4} \frac{A(x)}{\omega} \frac{\partial A(x)}{\partial x} - \frac{1}{2\omega} \frac{A^2(x)}{r(x)} \frac{\partial r(x)}{\partial x} + A(x)\cos\omega t. \tag{2.7}$$

In the case of a sphere $r(x) = R \sin x/R$, $A(x) = {}^3/_2 B \sin(x/R)$, so that for $y > \delta^+$ the equation for u_0 becomes

$$u_0 = -1.4 B^2 (\omega R)^{-1} \sin(2x/R). \tag{2.8}$$

A solution of the system (2.1), (2.2) was obtained by Roy [5], but this contains an error due to the wrong choice of sign in the expression for the transverse velocity component.

3. Let us calculate the distribution of concentration over the surface of the sphere for $s/R \ll 1$. As the stationary flow is directed towards the equator of the sphere (Eq. (2.8)), we must write the diffusion equation in the o'xy coordi-

nate system (Fig. 1)

$$\frac{\partial m}{\partial t}+u\frac{\partial m}{\partial x}+v\frac{\partial m}{\partial y}=D\frac{\partial^2 m}{\partial y^2}\,. \tag{3.1}$$

By analogy with the velocity field, let us represent the concentration field in the form

$$m=m_0+m', \tag{3.2}$$

where m_0, m' are the stationary and pulsating concentration components, respectively.

Substituting (3. 2) into (3. 1), and averaging Eq. (3. 1) term by term according to Reynolds's, we obtain

$$u_0\frac{\partial m_0}{\partial x}+v_0\frac{\partial m_0}{\partial y}=D\frac{\partial^2 m_0}{\partial y^2}-\left\langle u'\frac{\partial m'}{\partial x}\right\rangle-\left\langle v'\frac{\partial m'}{\partial y}\right\rangle. \tag{3.3}$$

The boundary conditions associated with (3. 3) are

$$m_0=m^* \quad \text{at} \quad y=0, \qquad m_0=0 \quad \text{at} \quad y=\infty.$$

The ratio of the momentum and diffusion boundary-layer thicknesses is

$$\frac{\delta^+}{\delta^-}\approx\frac{s}{R}P^{1/2} \quad \text{at} \quad n=\frac{1}{2}\left(\delta\approx\left(\frac{DR}{u_0}\right)^{1/2}\right).$$

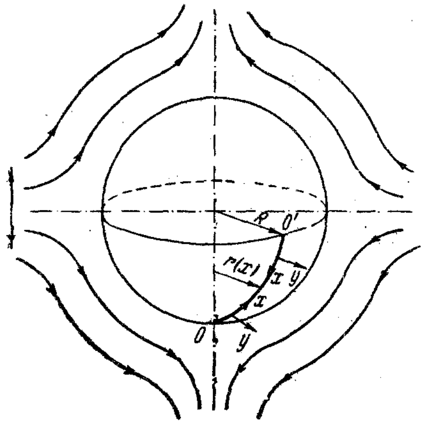

Fig. 1

When $P \leq 1$ and $s/R \ll 1$ the momentum boundary layer lies wholly inside the diffusion boundary layer. Neglecting the diffusion resistance of the momentum boundary layer, we obtain, in the o'xy system (Fig. 1), the results

$$u_0=1.4\frac{B^2}{\omega R}\sin\frac{2x}{R}, \qquad v_0=-\int_0^y\frac{\partial(u_0r)}{r\partial x}dy,$$
$$u'=\frac{3}{2}B\sin\frac{x}{R}\cos\omega t, \tag{3.4}$$

$$v'=\frac{3B}{R}\sin\frac{x}{R}\Big(y\cos\omega t-$$
$$-\frac{1}{2}\delta^+\cos\omega t-\frac{1}{2}\delta^+\sin\omega t\Big). \tag{3.5}$$

The variable radius of the sphere in the new system of coordinates is given by $r = R\cos(x/R)$.

Equation (3. 3) contains the unknown value of the pulsating component of the concentration m' in the diffusion boundary layer. Using Lighthill's theory [6], m' can be determined from

$$m'=-\int u'\frac{\partial m_0}{\partial x}dt-\int v'\frac{\partial m_0}{\partial y}dt \qquad (y>\delta^+). \tag{3.6}$$

Using (3. 6) and (3. 5), one can calculate the pulsating mass transfer terms

$$\left\langle u'\frac{\partial m'}{\partial x}\right\rangle+\left\langle v'\frac{\partial m'}{\partial y}\right\rangle=-\frac{9}{4}\frac{B^2\delta^+}{\omega R^2}\left[\frac{1}{2}\cos^2\frac{x}{R}-\sin^2\frac{x}{R}\right]\frac{\partial m_0}{\partial y}, \tag{3.7}$$

which are δ^+ times smaller than the convective mass transfer terms in (3. 3). If we neglect the pulsating mass transfer terms, Eq. (3. 3) can be reduced to the form

$$\frac{\partial m_0}{\partial\theta}=D\frac{\partial^2 m_0}{\partial\psi^2} \qquad \left(\theta=\int_0^x u_0r^2dx, \quad \psi=\int_k^x u_0r\,dy\right), \tag{3.8}$$

with the boundary conditions

$$m = m^* \quad \text{at} \quad \psi = 0, \quad m_0 = 0 \quad \text{at} \quad \psi = \infty, \quad m_0 = 0 \quad \text{at} \quad \psi = 0, \ \theta = 0. \tag{3.9}$$

The solution to (3.8), taking into account (3.9), is the well-known function

$$m_0 = m^* \left(1 - \operatorname{erf} \frac{\psi}{2\sqrt{D\theta}}\right). \tag{3.10}$$

In accordance with (3.10) the dimensionless mass transfer coefficient

$$N = 1.89 \frac{B\Phi}{\sqrt{\omega D}} \quad \left(\Phi = \frac{\cos^2 \varphi}{\sqrt{1 + \cos^2 \varphi}}, \quad \varphi = \frac{x}{R}\right). \tag{3.11}$$

Taking into account the pulsating mass transfer terms in (3.3), one obtains

$$N = 1.89 \frac{B\Phi}{\sqrt{\omega D}} \frac{\exp\left[-(\alpha\Phi)^2\right]}{(\alpha\Phi)} \tag{3.12}$$

$$(\alpha = 0.95 \; S/RP)^{1/2}.$$

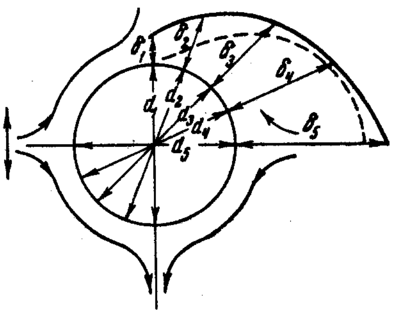

Fig. 2

Regarding Eq. (3.12), it should be noted that the intensification of mass transfer due to velocity pulsation is negligibly small, i.e., for all practical purposes one may use Eq. (3.11), which, averaged over the sphere, yields

$$N_d = 1.3B / \sqrt{\omega D}. \tag{3.13}$$

4. The aim of the experimental investigation of mass transfer from a sphere in an acoustic field was to establish the dependence of the dimensionless mass transfer coefficient on the parameters of the acoustic field for the case when $s/R \ll 1$ and $\lambda/2\pi R \geq 1$ (when $\lambda/2\pi R < 1$ second-order effects appear – reflection of the sound wave from the body surface, formation of a "sound shadow," etc.). The experiments were carried out with camphor balls evaporating into a field of standing acoustic waves of frequency 11.5 and 18 kc and intensity 150-163 db (0.1-2.15 W/cm^2). The sound generator was electrodynamic emitter, which made it possible to produce harmonic oscillations of rigorously controlled frequency, determined by the geometric dimensions of the emitter.

To avoid effects of porosity, the balls were prepared by immersing a spherical metal core for a short time in the vapor of boiling camphor. The balls thus produced had a diameter of 3, 5, 6, or 10 mm with a surface layer of camphor 0.1-0.2 mm thick. Microscopic examination of ball surfaces did not reveal any cracks or pores.

To investigate the role of secondary flow in mass transfer, we carried out experiments to determine local mass transfer coefficients. A camphor ball, welded to a special holder, was mounted on the object state of a IZA-2 horizontal comparator, operating with a MOV-1-15 ocular screw micrometer (the total error of linear measurement was 2-3 μ).

The diameter of the ball was measured at five positions $d_1 - d_5$ (Fig. 2) before the after evaporation in the acoustic field. Figure 2 presents the results of the local mass transfer measurements. Analyzing the curve which represents the change in ball diameter over the surface, one sees that the "wear" was highest at those points of the ball surface that formed the forward stagnation points of the secondary flow, and lowest at the points where the secondary flow left the surface. The broken line, constructed from Eq. (3.11), is in satisfactory agreement with the experimental results (continuous line). Thus, it appears that the secondary flow is the main factor governing the intensity of mass transfer from a surface in an acoustic field. Experiments to determine the over-all mass transfer coefficients were carried out by the weighing method. The outer geometric surface of the ball was taken to be the surface of evaporation. Its diameter was measured on the comparator.

The intensity of the acoustic oscillations was measured by means of an AZ-2 acoustic probe, connected to a VZ-2A voltmeter and an oscilloscope.

The spherical barium titanate sensor of the acoustic probe and the camphor ball with its holder were mounted over the sound emitter in a special support, which made it possible to hold the ball at an antinode of the sound wave.

The change in weight of the balls was measured by weighing on an ADV-200 analytic balance before and after the experiment.

The air temperature at the evaporating surface was measured by a chromel-copel* thermocouple. The values of

*Cu-Ni alloy.

the diffusion coefficient of camphor vapor in air and its saturation pressure were taken from [7, 8].

Figure 3 shows the results of the experiments, represented in the form of a relation between the diffusion Nusselt number N_D and the dimensionless group $B^* = (B^2/\omega D)^{1/2}$, which is the Peclet number referred to the velocity of the secondary flow. The figure also shows the results of theoretical calculations: curve (1) is based on Eq. (3.13), curve (2) represents the equation $N_d = 2 + 1.3B^*$. The experimental data correspond to the following values of f (kc) and d (mm): points 1 – (18, 6); 2 – (18, 3.5); 3 – (11.5, 10); 4 – (11.5, 6); 5 – (11.5, 3.5).

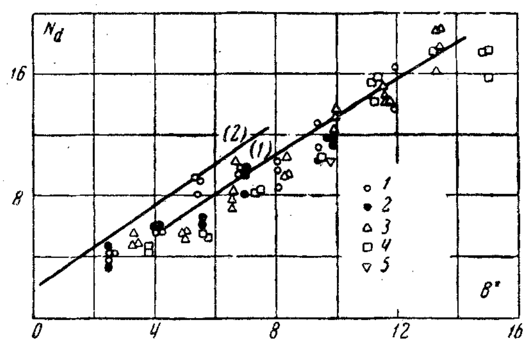

Fig. 3

As can be seen from the figure, when $B^* > 3$ the dimensionless mass transfer coefficient can be determined from Eq. (3.13), while for $B^* < 3$ the equation $N_d = 2 + 1.3B^*$ is better.

Thus, in an acoustic field with $s/R < 1$ and $\lambda/2\pi R > 1$, the dimensionless mass transfer coefficient for a sphere is independent of the diameter of the sphere, increases with increasing sould intensity, and is inversely proportional to the square root of the frequency.

5. The case $s/R \gg 1$ is always realized in pulsating combustion chambers. According to Reynst's date [9], the amplitude of particle displacement is 0.5-1 m, i.e., certainly many times greater than the dimension of the flame particles. Under these conditions the mass transfer process can be regarded as quasi-stationary. Mass transfer relations obtained for a steady-state process are valid at each instant for the quasi-stationary process. If the velocity of the gas varies according to a harmonic law, then using Eckert's equation $N_d = 0.37 R_d^{0.6} P^{1/3}$ (where R_d is the Reynolds number) [10] we obtain the time average

$$N_d = 0.259 P^{1/3} (Bd/\nu)^{0.6} .$$

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